Homework 7 Solutions

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1. The Hubble Diagram.

(a) **From the following velocity-distance diagram, calculate the Hubble constant in units of km s^{-1} Mpc^{-1}.** The plot is graphical expression of Hubble's Law, which states $v = H_0 \cdot d$, or the recessional velocity (in km/s) of a galaxy is equal to its distance (in Mpc) times the Hubble constant. Thus, the Hubble constant is simply the slope of the line, which we can find to be,

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{(300 - 0) \text{ km s}^{-1}}{(4 - 0) \text{ Mpc}} = 75 \text{ km s}^{-1} \text{ Mpc}^{-1} \tag{1}$$

(b) **Please explain how the above diagram can be used as a basis for an estimate of the age of the universe.** For simplicity we assume that the universe has always expanded at the same rate (not quite true, but a pretty good approximation). Then if we consider a galaxy at a distance d and moving with velocity v, we know that at a time $t = v/d$ in the past, it would have been at zero distance from us. This is simply using the formula for motion at a constant velocity, $d = vt$. Comparing this formula with Hubble's law, $v = H_0 d$, we find that this time is given by $t = H_0^{-1}$. Since we're assuming that the universe has always been expanding at the same rate, this time will be the same for galaxies at every distance from us. That is, at some time in the past, every galaxy was at zero distance from every other galaxy. We call that point in time the Big Bang.

(c) **Using the information in the diagram above, what is the expansion age of the universe?** From above, the expansion age of the universe is given by $t = H_0^{-1}$. But, the units of H_0 are given in km s^{-1} Mpc^{-1}, so we need to convert those units to units of time to get our answer. Since km and Mpc are both units of distance we can mutually cancel them by multiplying by the conversion factor between them.

$$t = \frac{1}{H_0} = \frac{\text{s Mpc}}{75 \text{ km}} = \frac{\text{s Mpc}}{75 \text{ km}} \cdot \frac{3 \times 10^{19} \text{ km}}{\text{Mpc}} = 4 \times 10^{17} \text{ s}. \tag{2}$$

Noting that there are 3×10^7 seconds in a year, we can convert this to years,

$$4 \times 10^{17} \text{ s} \cdot \frac{\text{year}}{3 \times 10^7 \text{ s}} = 1.33 \times 10^{10} \text{ years} = 13.3 \text{ Gyr}. \tag{3}$$

A common mistake on this problem was to combine the little "h" factor from class with the value of 75. The problem is that the little "h" factor is only used when assuming $H_0 = 100 \cdot h$ km s^{-1} Mpc^{-1}, and is used to describe the difference of the actual value from 100. Astronomers have constrained the little "h" to be $0.6 < h < 0.8$, which is the same as saying $60 < H_0 < 80$. In our case, $H_0 = 75$ has *already* incorporated the h value, in particular $h = 0.75$, which falls nicely in the above range. So writing something like $75 \cdot h$ is actually multiplying by h twice, thereby making H_0 too small.

(d) **Hubble originally determined an expansion rate of** 500 **km s^{-1} Mpc $^{-1}$. (1) What was his expansion age for the universe?** Just like above we write

$$t = \frac{1}{H_0} = \frac{\text{s Mpc}}{500 \text{ km}} = \frac{\text{s Mpc}}{500 \text{ km}} \cdot \frac{3 \times 10^{19} \text{ km}}{\text{Mpc}} \cdot \frac{\text{year}}{3 \times 10^7 \text{ s}} = 2 \times 10^9 \text{ years} = 2 \text{ Gyr} \tag{4}$$

(2) Why was this age a problem?. This age was a problem because scientists studying the decay of radioactive elements in certain stars and in the Earth's crust determined that those stars and the Earth itself were *older* than 2 Gyr. This was embarassing. It turned out that the Cepheid variable stars Hubble was using as his standard candles were more complicated than was commonly assumed. After re-calibrating the Cepheids, the estimate for H_0 was lowered, thus making the estimate of the expansion age comfortably larger than the age of the Earth and the ages of the oldest stars.